Representations of Algebras and Related Combinatorics

March 12-13, 2022

Abstracts

Quasi-cluster algebras

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Quasi-cluster algebras were defined in 2015 by Dupont and Palesi and are cluster algebras arising from non-orientable surfaces. In this talk, we will first give an introduction to these quasi-cluster algebras and list some of their properties (finite-type classification, skein relations, among others). Then, we will count the number of triangulations for the finite type and make a connection with gentle algebras.

Vanishing and non-vanishing of Massey products for curves

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In 2014, Minac and Duy Tan showed that triple Massey products vanish for the absolute Galois group of any field F. In 2019, Harpaz and Wittenberg showed that this remains true for all higher Massey products in the case when F is a number field. The first natural case to consider beyond fields is that of Massey products over curves over fields. I will discuss some known and new vanishing and non-vanishing results in this case. The main tool is the representation theory of etale fundamental groups into Heisenberg groups. I will begin with background about Massey products, which first arose in topology, and about the relevant representation theory. This is joint work with Ted Chinburg and Jean Gillibert.

Mutations of reflections and existence of pseudo-acyclic orderings for type $\mathbb{A}_n$

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In a recent paper by K.-H. Lee, K. Lee and M. Mills, a mutation of reflections in the universal Coxeter group is defined in association with a mutation of a quiver. A matrix representation of these reflections is determined by a linear ordering on the set of vertices of the quiver. It was conjectured that there exists an ordering — which we call a pseudo-acyclic ordering — such that whenever two mutation sequences of a quiver lead to the same labeled seed, the representations of the associated reflections also coincide. We prove this conjecture for every quiver mutation-equivalent to an orientation of a type $\mathbb{A}_n$ Dynkin diagram by decomposing a mutation sequence into a product of elementary swaps and checking relations studied by Barot and Marsh.

This is a joint work with Blake Jackson, Kyu-Hwan Lee, and Kyungyong Lee.

Tensor Products of Leibniz Bimodules

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Leibniz algebras were introduced by Bloh and Loday as non-commutative analogues of Lie algebras. Many results for modules over Lie algebras have been proven to hold for bimodules over Leibniz algebras, but there are also several results that are not true in this more general context.

Let $\mathfrak{L}$ be a (left) Leibniz algebra. In general, the tensor product $M \otimes N$ of two $\mathfrak{L}$-bimodules M and N is not an $\mathfrak{L}$-bimodule, but by dividing out a certain subspace of $M \otimes N$ that is invariant under the left and right $\mathfrak{L}$-actions one obtains again an $\mathfrak{L}$-bimodule (which I call the truncated tensor product of M and N). In many cases, but not always, such a truncated tensor product is non-zero. In my talk I will report on some structural properties of truncated tensor products of Leibniz bimodules.

This is joint work with Friedrich Wagemann.

Triangulations and maximal almost rigid representations

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Let kQ/I be a finite-representation type gentle algebra. We present a bijection between certain triangulations and a class of nicely-behaved modules of kQ/I called *maximal almost rigid (mar).* We define an oriented flip graph of the mar modules and conjecture that it is an acyclic graph. Our results generalize the hereditary type $\mathbb{A}$ case, where the mar modules correspond to triangulations of a polygon and the oriented flip graph defines a poset which is a Tamari or Cambrian lattice.

This talk is based on joint work in progress with Emily Barnard, Raquel Coelho Simes, and Ralf Schiffler.

Homological approximations in persistence theory

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We define a class of invariants, which we call *homological invariants,* for persistence modules over a finite poset. We show that both the dimension vector and the rank invariant are homological invariants. We then provide an explicit example of a homological invariant which is finer than the rank invariant.

This is based on joint work with Benjamin Blanchette and Thomas Brüstle.

The Jacobian, reflection arrangement, and discriminant for reflection Hopf algebras

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Let $\Bbbk$ be an algebraically closed field of characteristic zero. When H is a semisimple Hopf algebra that acts inner faithfully and homogeneously on an Artin-Schelter regular algebra A so that A^H is also Artin-Schelter regular, we call H a reflection Hopf algebra for A; when $H = \Bbbk[G]$ and $A = \Bbbk[x_1, \ldots, x_n]$ then H is a reflection Hopf algebra for A if and only if G is a reflection group. We show that there exist notions of the Jacobian, reflection arrangement, and discriminant that extend the definitions used for reflection groups actions on polynomial algebras to this noncommutative setting.

Distribution of bricks over tame algebras (Brick discrete vs. Brick continuous)

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For an algebra A, let brick(A) denote the set of all isomorphism classes of bricks over A. That is, for every M in brick(A), all nonzero A-homomorphisms from M to M are invertible. Bricks are known to play pivotal roles in the combinatorial, lattice theoretical, homological and geometric aspects of representation theory of A. In this talk, we discuss an open conjecture on the behavior of bricks and primarily focus on tame representation-infinite algebras. After a brief recollection of some fundamental properties, we introduce the notion of *generic brick* to state a stronger version of our conjecture over tame algebras. For (special) biserial algebras, we verify the stronger conjecture. Finally, we leave some remarks on the more general setting.

This is a report on joint work in progress with Charles Paquette.

Triangulations of flow polytopes, ample framings, and gentle algebras

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We study directed acyclic graphs together with an ordering on the edges at each vertex called a framing. A result by Danilov, Karzanov, and Koshevoy shows a relation between framings and triangulations of flow polytopes coming from a graph. By placing some additional restrictions on the graphs we establish a connection between flow polytopes and representation theory of gentle algebras. Then we apply results about tau-tilting posets to derive a shelling of these triangulations. Furthermore, in this case we prove that the flow polytopes are Gorenstein and h*-unimodal.

Noncrossing partitions of classical affine type

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Suppose W is a Coxeter group and c is a Coxeter element. The *noncrossing partitions* in W are the elements of the interval $[1, c]_T$ in the *absolute order* (the prefix order relative to the alphabet T of all reflections in W). The noncrossing partitions have representation-theoretic significance and applications to the theory of Artin groups. When W is a finite Coxeter group, $[1, c]_T$ is a lattice. In the classical finite types, there are planar diagrams for noncrossing partitions. When W is infinite, $[1, c]_T$ may not be a lattice. In the affine case, McCammond and Sulway embedded W into a larger group in which an analogous interval $[1, c]$ is a lattice (thus proving some longstanding conjectures for Euclidean Artin groups). Their key idea was to factor the *translations* in $[1, c]_T$. I'll discuss work in progress to construct planar diagrams for noncrossing partitions of classical affine types. The planar diagrams "know" how to factor translations, and thus yield new insights into the McCammond-Sulway construction.

This is joint work with Laura Brestensky.

Hammocks and arch bridge quiver for domestic string algebras

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The class of string algebras form a test subclass for various conjectures regarding tame algebras due to the possibility of explicit computations and are presented with the help of quivers and monomial relations. Gelfand and Ponomarev showed that there are two kinds of vertices of the Auslander-Reiten quiver of a string algebra – they are understood using certain walks on the quiver known as strings, and some cyclic strings known as bands. The *bridge quiver* associated with a string algebra comprises bands as vertices and some special strings called *bridges* as arrows and encodes useful information about certain algebraic invariants. Domestic string algebras are characterized by their acyclic bridge quivers. The collection of strings starting/ ending with a fixed vertex can be equipped with a total order relation; such orders are known as hammocks. Schröer proved that hammocks for domestic string algebras are discrete linear orders with finite Hausdorff rank. I will explain a new combinatorial technique of *terms* that links hammocks with an extension of the bridge quiver known as the *arch bridge quiver.*

A geometric model for syzygies over 2-Calabi-Yau tilted algebras

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We study a certain family of 2-Calabi-Yau tilted algebras, which we call *dimer tree algebras,* since they may be realized as quotients of dimer algebras on a disk. These algebras are defined by a quiver with potential whose dual graph is a tree, and they are generally of wild representation type. Given such an algebra B, we construct a polygon $\mathcal{S}$ with a checkerboard pattern in its interior that defines a category $\mathrm{diag}(\mathcal{S})$. The indecomposable objects are the 2-diagonals in $\mathcal{S}$, and its morphisms are certain pivoting moves between the 2-diagonals. We prove that the category $\mathrm{diag}(\mathcal{S})$ is equivalent to the stable syzygy category of the algebra B.

As a consequence, we conclude that the number of indecomposable syzygies is finite, and moreover the syzygy category is equivalent to the 2-cluster category of type $\mathbb{A}$. In addition, we obtain an explicit description of the projective resolutions, which are periodic. Finally, the number of vertices of the polygon $\mathcal{S}$ is a derived invariant and a singular invariant for dimer tree algebras, which can be easily computed from the quiver.

Combinatorial Aspects of Exceptional Sequences

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We construct a representation-theoretic bijection between rooted labeled forests with n vertices and complete exceptional sequences for the quiver of type $\mathbb{A}_n$ with straight orientation. The ascending and descending vertices in the forest correspond to relatively projective and relatively injective objects in the exceptional sequence. We conclude that every object in an exceptional sequence is either relatively projective or relatively injective or both. If time permits, we talk about a natural action of the extended braid group on rooted labeled forests and show that it agrees with the known action of the braid group on complete exceptional sequences.

This is joint work with Kiyoshi Igusa.

Littlewood–Richardson tableaux, socle tableaux, and tableau switching

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A short exact sequence of vector spaces equipped with nilpotent transformations defines a Littlewood–Richardson tableau. In recent work, Kosakowska and Schmidmeier defined another tableau associated to such a sequence of vector spaces, which they call the socle tableau.

They conjecture that the socle tableau of a short exact sequence is related to the Littlewood–Richardson tableau of the dual sequence by a combinatorial procedure known as tableau switching. We prove this conjecture.

This is joint work with Gabe Frieden and Steven Karp.

Singular equivalences of Morita type with level,
Gorenstein algebras, and universal deformation rings

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Let $\mathbf{k}$ be a field of arbitrary characteristic, let Λ be a Gorenstein $\mathbf{k}$-algebra, and let V be an indecomposable finitely generated non-projective Gorenstein-projective left Λ-module whose stable endomorphism ring is isomorphic to $\mathbf{k}$. In this article, we prove that the universal deformation rings $R(\Lambda, V)$ and $R(\Lambda, \Omega_\Lambda V)$ are isomorphic, where $\Omega_\Lambda V$ denotes the first syzygy of V as a left Λ-module. We also prove the following result. Assume that Γ is another Gorenstein $\mathbf{k}$-algebra such that there exists $\ell \geq 0$ and a pair of bimodules $({}_\Gamma X_\Lambda, {}_\Lambda Y_\Gamma)$ that induces a singular equivalence of Morita type with level ℓ (as introduced by Z. Wang) between Λ and Γ. Then the left Γ-module $X \otimes_\Lambda V$ is also Gorenstein-projective and the universal deformation rings $R(\Gamma, X \otimes_\Lambda V)$ and $R(\Lambda, V)$ are isomorphic.

Groupoids of 2-Calabi-Yau categories,
faithful derived actions and hyperplane arrangements

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Beginning with the fundamental work of Seidel and Thomas, there has been a great interest in the construction of faithful actions of various classes of groups on derived categories (braid groups, fundamental groups of hyperplane arrangements, mapping class groups). We will describe a general construction of this type starting from a categorical setting that is present in a broad range of situations in representation theory, algebraic geometry, cluster algebras, and topology. To each 2-Calabi-Yau triangulated category we associate a groupoid in an intrinsic homological way. It is a rich invariant of the category which is a far-reaching generalization of the Deligne groupoid of a hyperplane arrangement. We will then present a construction of categorical representations of these goupoids by derived equivalences associated to all Frobenius models of the Calabi-Yau categories. Under natural assumptions, we will prove that the actions are faithful.

This is a joint work with Peter Jørgensen (Aarhus University).

Last modified: March 7, 2022, by Markus Schmidmeier